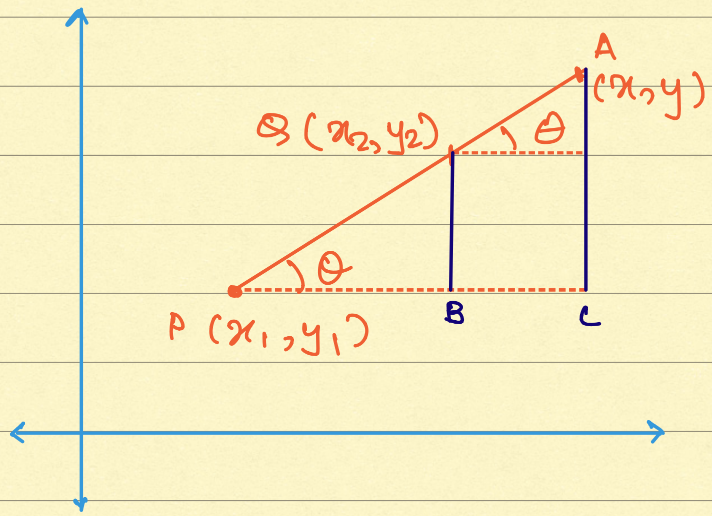
EQUATION OF ST. LINES

① Two-point form :

Line passing through two points



$l(PB) = x_2 - x_1$

$l(PC) = x - x_1$

$$\tan\theta = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_1}{x - x_1} = m$$

$$\therefore \; y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

② Slope-Point Form

Given : m $\quad P(x_1, y_1)$

$$y - y_1 = m(x - x_1)$$

③ **Determinant Form** (Not that frequently used)

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

$P(x, y)$

$A(x_1, y_1)$, $B(x_2, y_2)$

④ **Slope-Intercept Form**

Given: m & $(0, c)$

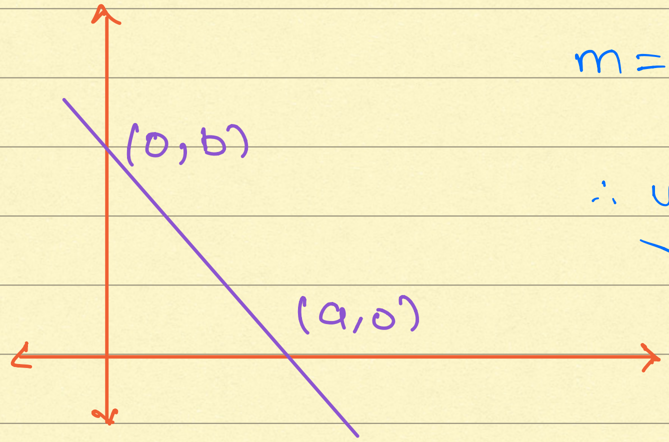
$\therefore y - c = m(x - 0)$

$y = mx + c$

⑤ **Double-Intercept Form**:

x-intercept: $(a, 0)$

y-intercept: $(0, b)$



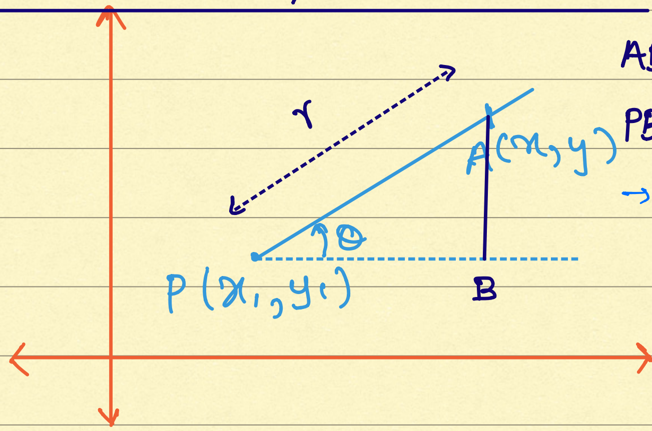
$m = -\dfrac{b}{a}$

$\therefore y = -\dfrac{b}{a}x + b$

$\dfrac{y}{b} = -\dfrac{x}{a} + 1$

$\Rightarrow \dfrac{x}{a} + \dfrac{y}{b} = 1$

⑥ Parametric / Polar Form:

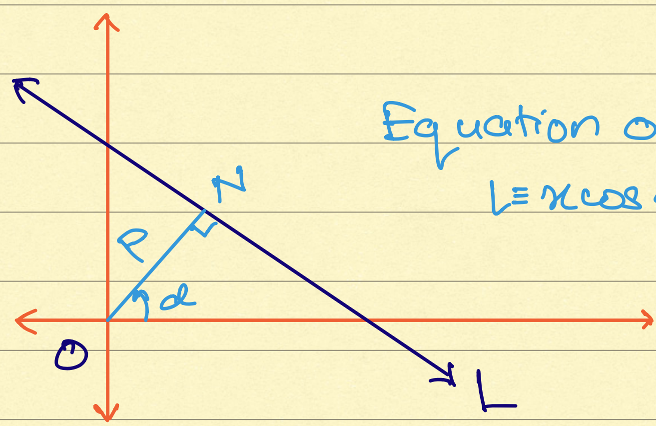


$AB = y - y_1 = r\sin\theta$

$PB = x - x_1 = r\cos\theta$

$\rightarrow \dfrac{x - x_1}{\cos\theta} = \dfrac{y - y_1}{\sin\theta} = r$

⑦ Normal Form:



Equation of the line:

$L \equiv x\cos\alpha + y\sin\alpha = p$

⑧ General Form:

$ax + by + c = 0 \quad \Rightarrow \quad y = -\dfrac{a}{b}x - \dfrac{c}{b}$

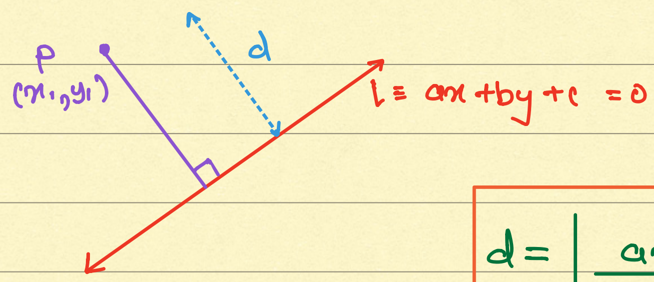
$\rightarrow$ Slope $= -\dfrac{a}{b}$, y- intercept $= -\dfrac{c}{b}$

$\rightarrow$ x- intercept $= -c/a$

$\perp^r$ Distance, Foot of $\perp^r$, Image

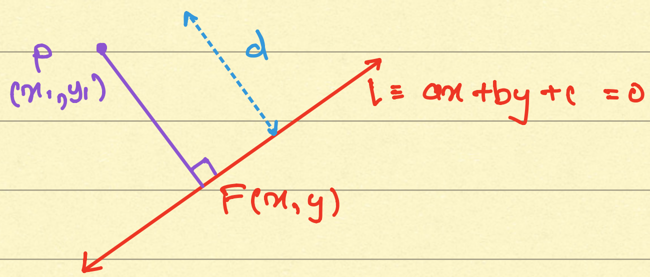
Let a given line L be $ax+by+c=0$
& a point $P(x_1, y_1)$

① Perpendicular Distance of P from L:



$$\boxed{d = \left| \frac{ax_1+by_1+c}{\sqrt{a^2+b^2}} \right|}$$

② Co-ordinates of foot of perpendicular



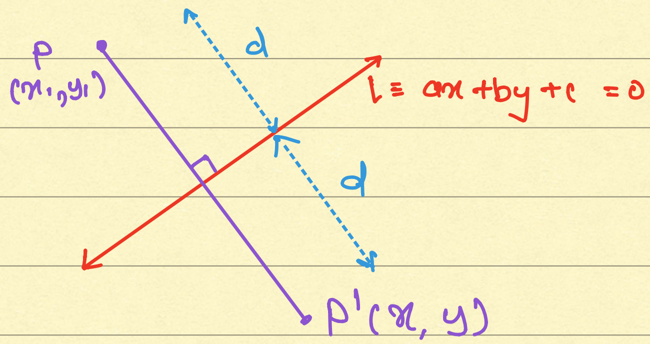
F: foot of perpendicular : (x, y)

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{-(ax_1+by_1+c)}{a^2+b^2}$$

x-coordinate y-coordinate

of foot   of foot

③ Image of a point

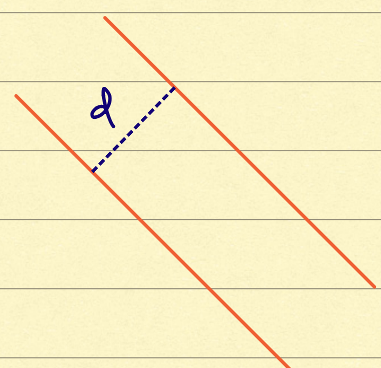


P': Image of point P in (x, y)

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$$

x-coordinate of image y-coordinate of image

④ Distance between parallel lines



$$d = \left| \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right|$$

$L_1 \equiv ax + by + c_1$

$L_2 \equiv ax + by + c_2$

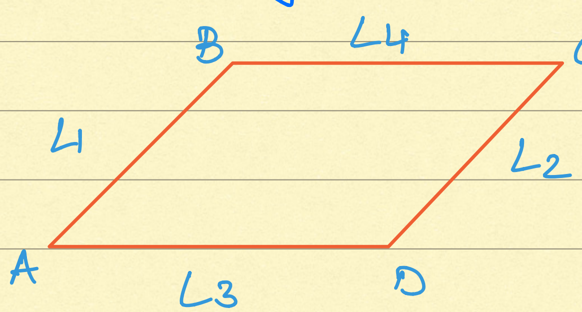
* Equations of parallel lines only differ

by the constant term

⑤ Area of a parallellogram formed by two pairs of parallel lines

Lines: $L_1 \equiv y = m_1 x + c_1$, $L_2 \equiv y = m_1 x + c_2$

$L_3 \equiv y = m_2 x + d_1$, $L_4 \equiv y = m_2 x + d_2$



$$\boxed{A(\square ABCD) = \left| \frac{(c_1 - c_2)(d_1 - d_2)}{m_1 - m_2} \right|}$$

* Note: To use this formula, lines must be in $y = mx + c$ form.
If the are given in general form, first convert them to slope - point form

Question :

① Find the value of x if the distance b/w $A(x, -1)$ & $B(3, 2)$ is 5

② $A(6,-1)$, $B(1,3)$ $C(x,8)$. Find x if $AB = BC$

③ Find the co-ordinates of the point which divides the line segment $A(6,3)$ $B(-4,5)$ in the ratio $3:2$

i) internally ii) externally

④ $A(3,7)$, $B(9,2)$. B divides AP in the ratio $4:7$ internally. Find P.

⑤ Find λ if area of ΔABC is 10 unit^2 $A(1,2)$ $B(2,-3)$ $C(\lambda, 2\lambda)$

⑥ Vertices of a quadrilateral are $A(6,3)$, $B(-3,5)$ $C(4,-2)$ $D(x,3x)$

Find x if $A(\Delta ABC) = 2\,A(\Delta DBC)$

Solutions :

① $(x-3)^2 + (3^2) = 25$

$\Rightarrow (x-3)^2 = 16 \Rightarrow x-3 = 4$ or $x-3 = -4$

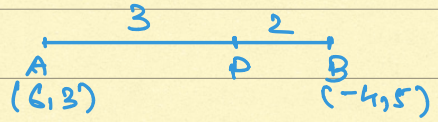
$x = 7$ or $x = -1$

② $AB = BC \Rightarrow (6-1)^2 + (-1-3)^2 = (x-1)^2 + 25$

$\Rightarrow 4^2 = (x-1)^2$

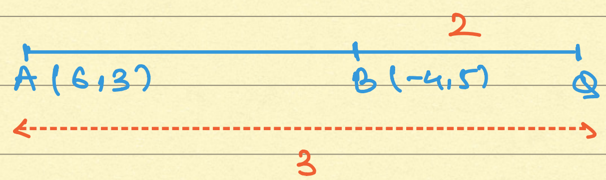
$\Rightarrow x = 5$ or $x = -3$

③ Internal division:



$$P(x,y) = \left(\frac{-12+12}{5}, \frac{15+6}{5} \right) = \left(0, \, 21/5\right)$$

External division:



$$Q = \left(\frac{3(-4) - 2(6)}{1}, \frac{3(5) - 2(3)}{1} \right) = (-24, 9)$$

④ $$B(9,2) = \left(\frac{4x+21}{11}, \frac{4y+49}{11} \right)$$

$$\Rightarrow x = \frac{78}{4}, \quad y = -\frac{27}{4}$$

⑤ $$10 = \frac{1}{2} \left\| \begin{vmatrix} 1 & 2 & 1 \\ 2 & -3 & 1 \\ \lambda & 2\lambda & 1 \end{vmatrix} \right\|$$

$$\Rightarrow 20 = |7\lambda - 7|$$

$$\Rightarrow \lambda = -13/7 \text{ or } \lambda = 27/7$$

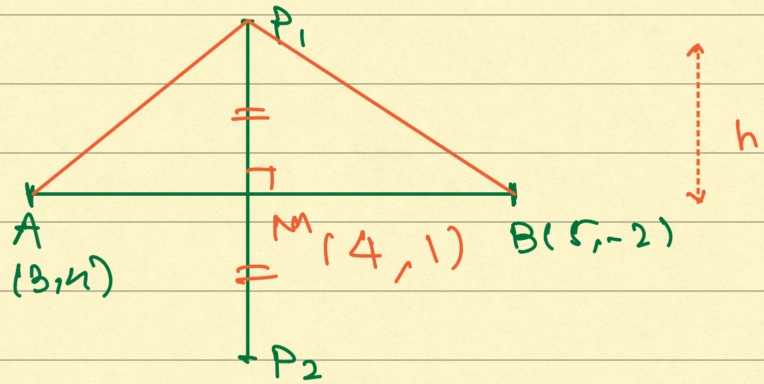
⑥ $\frac{1}{2}\left|\begin{vmatrix} 6 & 3 & 1 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{vmatrix}\right| = \left|\begin{vmatrix} x & 3x & 1 \\ -3 & 5 & 1 \\ 4 & -2 & 1 \end{vmatrix}\right|$

$\Rightarrow \frac{1}{2}|49| = |28x - 14|$

$\Rightarrow x = 11/8 \quad \text{or} \quad x = -3/8$

Q7) If A(3,4) & B(5,−2), find possible coordinates of P if PA = PB & A(APB) = 10 unit²

Ans) : $AB = \sqrt{2^2 + (6^2)} = \sqrt{40} = 2\sqrt{10}$ — ①



$\because A(\triangle APB) = 10, \quad \frac{1}{2}(AB)h = 10$

$\Rightarrow h = \sqrt{10}$

$\Rightarrow m_{AB} = \frac{-6}{2} = -3$

$\therefore m_{P_1P_2} = 1/3 \quad (\because P_1P_2 \perp AB)$

$P_1M = P_2M = \sqrt{10}$

$\therefore$ By using polar co-ordinates:

$P_1 = (4 + \sqrt{10}\cos\theta,\ 1 + \sqrt{10}\sin\theta)$

$P_2 = (4 - \sqrt{10}\cos\theta,\ 1 - \sqrt{10}\sin\theta)$

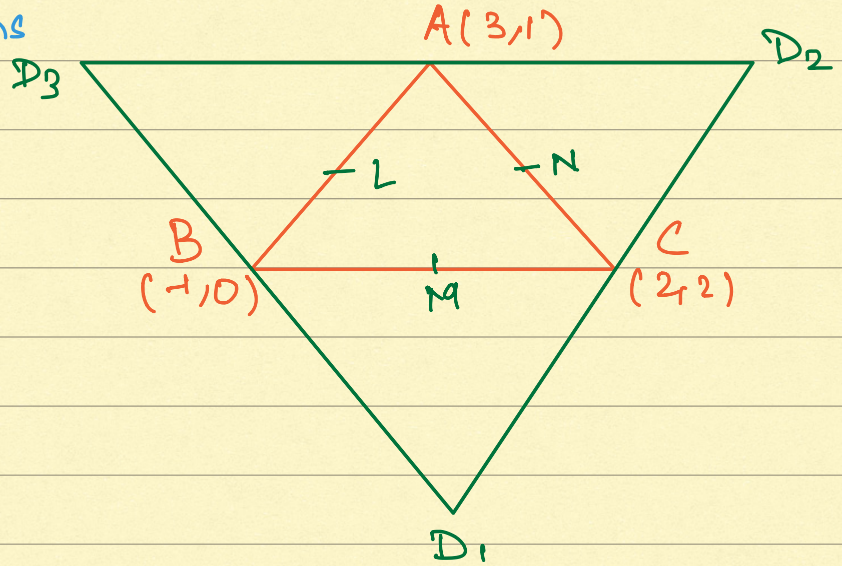
$\because \tan\theta = 1/3,\quad \cos\theta = 3/\sqrt{10},\ \sin\theta = 1/\sqrt{10}$

(use pythagoras)

$\therefore P_1 \equiv (7, 2) \qquad P_2 \equiv (1, 0)$

Q8) 3 vertices of a parallellogram are $(-1,0)$, $(3,1)$ & $(2,2)$. Find the possible co-ordinates of the 4th vertex.

Ans



- There are 3 possible values of D

for D_1, M is midpoint of AD_1

and BC.

$M \equiv (1/2, 1)$

$\therefore D_1 \equiv (-2, 1)$

- For D_2, N is the mid point of BD_2 & AC
- $N \equiv (5/2, 3/2)$
- $D_2 \equiv (6, 3)$

- Similarly, $L \equiv (1, 1/2)$
  $D_3 \equiv (0, -1)$

9) Check if (1,1), (2,3), (3,5) are collinear. If yes, find:

1. Eq^n of line joining them
2. slope
3. x, y intercepts
4. $\perp^r$ distance from origin
5. Foot of $\perp^r$ and image of origin

Ans: Line joining (1,1), (2,3):

$y - 1 = 2(x - 1)$

$\Rightarrow 2x - y - 1 = 0$

$\Rightarrow m = 2$

$\Rightarrow x, y$ intercepts: $x: 1/2$, $y: -1$

$\perp^r$ distance from origin:

$$d = \left| \frac{-1}{\sqrt{5}} \right| = 1/\sqrt{5}$$

Foot of $\perp^r$ from origin:

$$\frac{x-0}{2} = \frac{y-0}{-1} = \frac{-(-1)}{5}$$

$\Rightarrow x = 2/5,\ y = -1/5$ $\qquad F \equiv (2/5, -1/5)$

Image of origin:

$$\frac{x-0}{2} = \frac{y-0}{-1} = \frac{-2(-1)}{5}$$

$\Rightarrow x = 4/5,\ y = -2/5$

$P \equiv (4/5, -2/5)$

(10) Find the slope of the line with the following inclinations:

(1) $0°$ (2) $90°$ (3) $120°$ (4) $45°$ (5) $60°$ (6) $30°$

Ans) 0 ND $-\sqrt{3}$ 1 $\sqrt{3}$ $1/\sqrt{3}$

(11) Find the equations of the following lines:

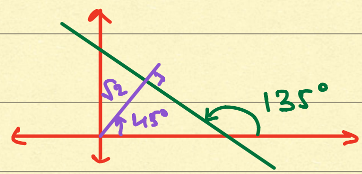
i) slope $= -1$, y-intercept $= 4$

ii) $\perp^r$ distance of $\sqrt{2}$ from the origin & making an angle of 135° with the x-axis

iii) Passing through $(3,4)$ with sum of intercepts $= 14$

Sol^n: i) $y = -x + 4$

ii)



$$\frac{x}{\sqrt{2}} + \frac{y}{2} = \sqrt{2}$$

$$\Rightarrow x + y = 2$$

iii) $$\frac{x}{a} + \frac{y}{14-a} = 1$$

Passes through $(3,4)$

$$-\quad \frac{3}{a} + \frac{4}{14-a} = 1$$

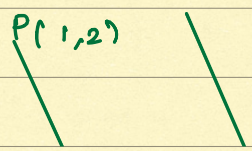
$$-\quad 42 - 3a + 4a = a(14-a)$$

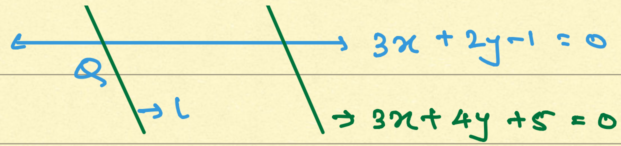
$$-\quad a^2 - 13a + 42 = 0 \Rightarrow a = 6 \text{ or } a = 7$$

$$-\quad l \equiv \frac{x}{6} + \frac{y}{8} = 1, \quad \frac{x}{7} + \frac{y}{7} = 1$$

(12) Find the distance of $P(1,2)$ from line $3x + 2y - 1 = 0$ measured along the line $3x + 4y + 5 = 0$

Ans)





- Line passing through P(1, 2) $\parallel^{el}$ to $3x+4y+5=0$

is: $3x+4y+k=0$

$\Rightarrow 3(1)+4(2)+k=0$

$\Rightarrow k=-11$

$\therefore l \equiv 3x+4y-11=0$

Intersection of $3x+4y=11$

& $3x+2y=1$

$\Rightarrow y=5,\ x=-3 \Rightarrow Q \equiv (-3, 5)$

$PQ = 5$

(13) Find the angle b/w the lines:

i) $2x-3y+4=0$ & $3x+4y-5=0$

ii) $3y-4=0$ & $4y-x+2=0$

iii) $x-8=0$ & $2x+y+3=0$

Ans : i) $\tan\theta = \left|\dfrac{m_1-m_2}{1+m_1m_2}\right|$ Here, $m_1 = 2/3$, $m_2 = -3/4$

$= \left|\dfrac{2/3-(-3/4)}{1+\left(\frac{2}{3}\right)\left(-\frac{3}{4}\right)}\right|$

$= \dfrac{17/12}{1/2} = 17/6 \Rightarrow \theta = \tan^{-1}(17/6)$

ii) $\tan\theta = \left|\dfrac{m_1-m_2}{1+m_1m_2}\right|$ Here $m_1 = 0$, $m_2 = 1/4$

$$\tan\theta = \left|\frac{1/4 - 0}{1+0}\right| = \frac{1}{4}$$

iii) Here m_1 is undefined,

$m_2 = -2$

$$\tan\theta = \lim_{m_1 \to \infty} \left|\frac{m_1 + 2}{1 - 2m_1}\right|$$

As $m_1 \to \infty$, 1 in denominator

& 2 in numerator

can be neglected

$$\therefore \tan\theta = \frac{m_1}{2m_1} = 1/2$$

(14) Find the distance b/w point & line:

i) $P = (3,3)$, $L \equiv x - 2 = 0$

ii) $P = (2,1)$ $l \equiv 12x - 5y + 9 = 0$

(15) Find the distance b/w the lines:

$6x + 8y + 14 = 0$ & $3x + 4y + 5 = 0$

(16) Find the area of the square two of whose sides are:

$x + y + 1 = 0$

$x + y + 2 = 0$

(17) Find the $\perp^r$ distance, foot of $\perp^r$ & image of the point $(-1, 2)$ wrt the line $2x - 3y + 4 = 0$

(18) Find the area of the $||^{gm}$ formed by the lines :

$x - 3y + 18 = 0$
$9x - 5y + 8 = 0$
$9x - 5y - 14 = 0$
$x - 3y - 4 = 0$

(19) Find m such that

$3x + y + 2 = 0$
$2x - y + 3 = 0$
$x + my - 3 = 0$ are concurrent

(20) Find the locus of a point whose distance from $(-1, 0)$ is always 3 times its distance from $(0, 2)$

(21) Find the locus of a point whose distance from $P(1, 2)$ is the same as its distance from the y-axis.

(22) Find the locus & area of the locus of the point, sum of whose $\perp^r$ distance from x & y-axes is always 1.

(23) A rod of length 'l' (constant) slides b/w the axes. Find the locus of the foot of $\perp^r$ drawn from the origin wrt the rod

Solutions :

14) i) $L = x - 2 = 0$

$P(3, 8)$

$d = \left| \frac{3-2}{\sqrt{1}} \right| = 1$

ii) $12x - 5y + 9 = 0$

$P(2, 1)$

$d = \left| \frac{12(2) - 5(1) + 9}{13} \right|$

$= 28/13$

15) $3x + 4y + 7 = 0$

$3x + 4y + 5 = 0$

$d = \left| \frac{7-5}{\sqrt{5}} \right| = 2/\sqrt{5}$

16) Side length : $\left. \begin{array}{l} x+y+1=0 \\ x+y+2=0 \end{array} \right\}$ Distance b/w

$$S = \left| \frac{1}{\sqrt{2}} \right| \Rightarrow A = 1/2$$

17) $P \equiv (-1, 2)$ $\quad l \equiv 2x - 3y + 4 = 0$

$\perp^r$ distance :

$d = 4/\sqrt{5}$

Foot of $\perp^r$:

$$\frac{x+1}{2} = \frac{y-2}{-3} = \frac{-(\ \ -4\ \ )}{13}$$

$x = -5/13, \quad y = 14/13 \quad \therefore F \equiv (-5/13, 14/13)$

Image :

$$\frac{x+1}{2} = \frac{y-2}{-3} = \frac{-2(-4)}{13}$$

$x = \frac{-3}{13}, \quad y = 2/13$

18) Area of parallellogram :

$y = \frac{x}{3} + 6 \qquad y = \frac{9}{5}x + \frac{8}{5}$

$y = \frac{x}{3} - \frac{4}{3} \qquad y = \frac{9}{5}x - \frac{14}{5}$

$$\Delta = \left| \frac{(6 + 4/3)(8/5 + 14/5)}{9/5 - 1/3} \right|$$

$$= \left| \frac{(22/3)(22/5)}{22/15} \right| = 22$$

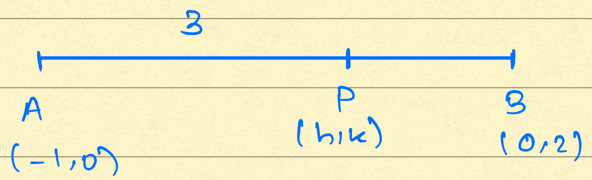
19) $\begin{vmatrix} 3 & 1 & 2 \\ 2 & -1 & 3 \\ 1 & m & -3 \end{vmatrix} = 3(3-3m) - 1(-9) + 2(2m+1) = 0$

$= 9 - 9m + 9 + 4m + 2 = 0$

$-5m + 20 = 0$

$\Rightarrow m = 4$

20) Let $P(h, k)$



$(h+1)^2 + k^2 = 9(h^2 + (k-2)^2)$

$\Rightarrow \quad 8h^2 + 8k^2 - 2h - 36k + 35 = 0$

21) Let $P \equiv (h, k)$

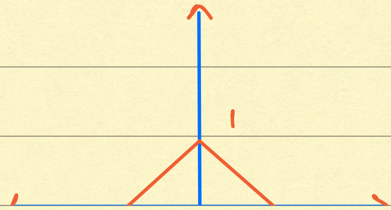
Distance from y-axis : $|h|$

$(h-1)^2 + (k-2)^2 = h^2$

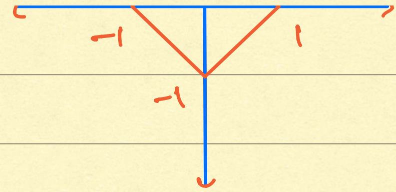
$k^2 - 4k + 4 - 2h + 1 = 0$

$y^2 - 4y - 2x + 5 = 0$

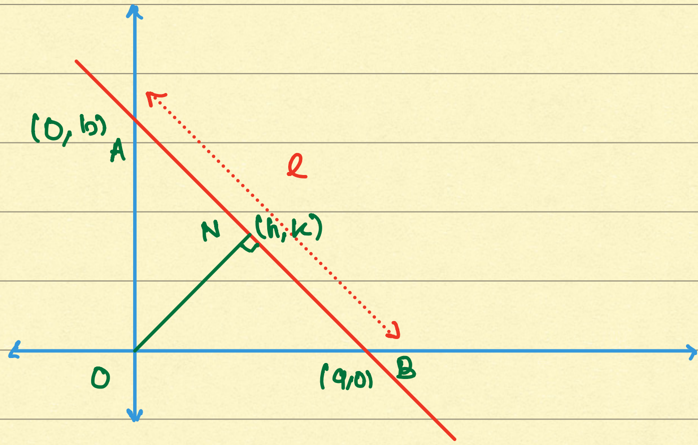
22) $|x| + |y| = 1$



$\Delta = (\sqrt{2})^2 = 2$



23)



$ON \perp AN$

$\Rightarrow \dfrac{k}{h} \cdot \left(\dfrac{k-b}{h}\right) = -1$

$k^2 - bk = -h^2$

$b = \dfrac{k^2+h^2}{k}$

$ON \perp NB$

$\dfrac{k}{h-a} \cdot \dfrac{k}{h} = -1$

$\Rightarrow k^2 = -h^2 + ah$

$\Rightarrow a = \dfrac{h^2+k^2}{h}$

$l^2 = a^2+b^2 = \dfrac{(h^2+k^2)^2}{h^2} + \dfrac{(h^2+k^2)^2}{k^2}$

$\Rightarrow l^2 = (h^2+k^2)^2 \left(\dfrac{1}{h^2} + \dfrac{1}{k^2}\right)$

$l^2 = \dfrac{(h^2+k^2)^3}{h^2k^2} \Rightarrow l^2h^2k^2 = (h^2+k^2)^3$

$\boxed{l^2x^2y^2 = (x^2+y^2)^3}$